# 2. WGS 84 AND ASSOCIATED REFERENCE FRAMES

## 2.1 General

Specifying the position of points on the earth's surface or in space is a complicated process due to many factors, some not thoroughly understood. The earth is in motion around the sun, the so-called fixed stars are not fixed in space but are also in motion, and at any point in time, the earth has an instantaneous astronomic axis of rotation, an instantaneous astronomic equator, and an instantaneous zero (astronomic) meridian. Since the reference frame of such an instantaneous earth is continually changing, it is unsuitable as a reference to which DMA-developed mapping, charting, and geodetic (MC&G) products and data sets can be related and must be replaced by the reference frame of a standard earth. Such a standard earth is rigid, has homogeneous density, rotates at a constant rate, and has a fixed (in time) spin axis. The Conventional Terrestrial System defined by the Bureau International de l'Heure (BIH) is the reference frame of such a standard earth.

## 2.2 Conventional Terrestrial System

### 2.2.1 Definition

The origin of the BIH-defined Conventional Terrestrial System (CTS) is the earth's center of mass. The Z-axis of the CTS is defined implicitly by the coordinates of the stations used by the BIH to determine polar motion from observational data. This mean pole, the Conventional Terrestrial Pole (CTP), replaces the Conventional International Origin (CIO), and is the origin of the polar motion parameters ($x_p$, $y_p$) derived and published by the BIH. The X-axis (Zero Meridian) of the CTS is defined implictly by the coordinates of the stations that provide observational data to the BIH for the determination of Universal Time (UT). This axis, the X-axis, is the point on the CTP's equator used by the BIH for deriving UT1. The Y-axis, measured in the plane of the CTP equator, 90° east of the X-axis, completes the reference frame.

The BIH-defined CTS, or BTS, (1984.0) was adopted as the WGS 84 Coordinate System or reference frame. The following Section discusses the practical realization of the WGS 84 Coordinate System (reference frame) by modifying the Naval Surface Warfare Center (NSWC) 9Z-2 Coordinate System.

### 2.2.2 Modification of the NSWC 9Z-2 Coordinate System

#### 2.2.2.1 General

During the development of WGS 72, a set of coordinates was adopted for the semi-permanent Doppler tracking network (TRANET) stations, and the coordinate system defined by these coordinates was used for computing precise ephemerides for the Navy Navigation Satellites through the period 1972-1984. The coordinate system was renamed from NWL 9D to NSWC 9Z-2 as coordinates for stations were refined. The changes were made in a manner intended to preserve the origin, orientation, and scale of the system, and there is no conclusive evidence that such station coordinate changes disturbed the system [2.1]. However, there were changes to the coordinate system arising from physical causes. Neglected ionospheric effects above second order cause the computed heights of Doppler stations to be higher during periods of high solar activity. The effect on station heights, which may reach two meters, is correlated with the 11-year solar activity cycle, with the latitude of the station with respect to the geomagnetic equator, with the local time of day of the observations, and with short period bursts in solar activity. The increase in the gradient of the ionosphere in the neighborhood of the stations may also affect the horizontal coordinates of the stations, particularly the latitude which is more likely to be subject to biases in the gradient. For example, changes of 1 meter in the computed latitude of the Mizusawa (Japan) Doppler station over a 10-year period may well arise from this source.

In addition to ionospheric effects on computed station positions, the stations move with the tectonic plates on which they are located [2.1]. For example, a substantial portion of the 1.5 meter change in the geodetic latitude of Australian Doppler stations with time may be due to the northward component of the motion of the Australian Plate.

Although it is conceivable that corrections could be derived for computed Doppler positions to account for higher order ionospheric and plate motion effects, the uncertainty of the corrections would be large when applied to an arbitrary geographic location, period of observation and satellite, but the corrections fairly small, when compared to other system errors. For example, the correlated orbit error due to errors in the earth gravitational model used to compute precise ephemerides can be considered to cause a distortion of the coordinate system. The precise amplitude of the distortion is not known, but it is believed to be approximately one meter with a wavelength of 500 kilometers [2.2]. Therefore, no attempt was made to model these effects in developing WGS 84 transformations which apply (as a result) to the average span of time over which the data were acquired.

#### 2.2.2.2 Origin Change

The origin of the NSWC 9Z-2 Coordinate System is known to be offset from its specified location at the earth's center of mass. Since laser station-to-satellite distances (ranges) can be calculated from laser range data with errors less than 0.5 meter, laser-derived geocentric coordinates are more accurate than Doppler geocentric coordinates derived in the NSWC 9Z-2 Coordinate System. Therefore, the Satellite Laser Ranging (SLR) Coordinate System (another geocentric system) is ideally suited for providing data to determine the displacement of the NSWC 9Z-2 Coordinate System origin from the earth's center of mass. This origin offset is commonly referred to as the Doppler Coordinate System Z-axis bias ($\Delta Z$).

Using various sets of SLR and Doppler-derived coordinates, the National Geodetic Survey (NGS) [2.3] determined that the equatorial plane of the NSWC 9Z-2 Coordinate System was north of the SLR Coordinate System equatorial plane by amounts ranging from $\Delta Z$ = 3.60 meters to $\Delta Z$ = 4.08 meters. A wider range of $\Delta Z$-values was obtained by the BIH from an analysis involving NSWC 9Z-2 Doppler coordinates and two sets of SLR-derived coordinates. The two SLR coordinate sets were provided to the BIH by the University of Texas Center for Space Research (UT/CSR; Austin, Texas) and the National Aeronautics and Space Administration/Goddard Space Flight Center (NASA/GSFC). The BIH investigation [2.4; Table 4, Page D-83] indicated that the Doppler Z-axis bias was either 4.36 meters or 5.61 meters depending on whether the analysis was based on SLR coordinates derived by UT/CSR or NASA/GSFC, respectively.

Based on the preceding and other related $\Delta Z$ investigations, DMA and NGS adopted a Z-axis bias of $\Delta Z$ = 4.5 meters for use with the NSWC 9Z-2 Coordinate System when developing WGS 84 and NAD 83. The adopted $\Delta Z$ value was assigned a one-sigma uncertainty of ±0.5 meter by DMA.

In a BIH analysis later than that reported on above, the BIH [2.5; Table 3, Page B-6] found the origins of the BIH-defined CTS and the SLR UT/CSR Coordinate System to be in agreement and reported the NSWC 9Z-2 Coordinate System origin offset from (displaced north of) the BTS origin by 4.90 meters. The most recent BIH annual report available, that for 1985 [2.6], lists a Z-axis bias of $\Delta Z$ = 4.73 meters. These last two $\Delta Z$ values, recorded in Table 2.1 with other previously discussed estimates, indicate that coincidence between the WGS 84 and BTS origins has been achieved to within the assigned uncertainty (±0.5m).

The neglect of a relativistic correction in the processing of Doppler data has been mentioned as a possible contributor to the uncertainty in Doppler-derived Z coordinates. Since a variety of satellites and arguments of perigee were used in establishing the Doppler system, it is unlikely that the lack of a relativistic correction is a significant factor in causing the bias. The Z-axis bias in the NSWC 9Z-2 Coordinate System is apparently real and unexplained.

For completeness, displacement of the NSWC 9Z-2 Coordinate System origin in the plane of the earth's equator (i.e., in the X and Y directions) should be addressed briefly. Investigations [2.3] - [2.6] indicate that the $\Delta X$ and $\Delta Y$ displacements are small, with considerable scatter, seldom attaining 0.5 meter in magnitude. Due to their small magnitude and the uncertainty associated with their determination, a $\Delta X$, $\Delta Y$ modification of the NSWC 9Z-2 Coordinate System origin was not considered to be technically appropriate.

In summary, the best estimate of the NSWC 9Z-2 Coordinate System Z-axis bias was taken by DMA and NGS [2.7] to be:

$$\Delta Z = 4.5 \text{ meters} \tag{2-1}$$

with a one-sigma uncertainty of ±0.5 meter. To implement this modification, a value of 4.5 meters is added to Z-coordinates that have been derived in the NSWC 9Z-2 Coordinate System.

#### 2.2.2.3 Re-Orienting in Longitude

One objective of the WGS 84 development process was to have the WGS 84 Zero Meridian (X-axis) coincident with the BIH-defined Zero Meridian (X-axis). Since the NSWC 9Z-2 Coordinate System was to be modified to become the WGS 84 Coordinate System, and the X-axis (zero meridian) of the NSWC 9Z-2 Coordinate System was known to be east of the BIH-defined Zero Meridian, a westward rotation of the NSWC 9Z-2 Coordinate System about its Z-axis was needed to bring the NSWC 9Z-2 zero meridian and the BIH-defined Zero Meridian (1984.0) into coincidence.

The longitude angle through which the zero meridian of the NSWC 9Z-2 Coordinate System must be rotated to achieve coincidence with the BIH-defined Zero Meridian has been a topic of considerable investigation and discussion. As part of their NAD 83 development effort, NGS investigated this topic using Very Long Baseline Interferometry (VLBI) data. In the analysis, NGS [2.3] used three different VLBI data sets and various strategies to develop several seven-parameter coordinate transformations between the NSWC 9Z-2 and VLBI Coordinate Systems. Two of the VLBI data sets were provided by NASA/GSFC. The third VLBI data set was obtained by NGS from York University (Department of Physics; Toronto, Canada). Results varied as a function of the VLBI data sets and strategies used in developing the seven-parameter coordinate transformations. The analysis indicated that the NSWC 9Z-2 zero meridian was east of the VLBI reference (zero) meridian by a longitude angle ranging:

- From 0.77 to 0.81 arc second [NASA/GSFC VLBI Data Set 1 (6 VLBI sites, Mark I data, data observation period = 1972-1978)]

- From 0.79 to 0.83 arc second [NASA/GSFC VLBI Data Set 2 (7 VLBI sites, Mark III data, data observation period = 1980-1981)]

- From 0.84 to 0.88 arc second [York University Data Set (3 VLBI sites, data observation period = 1977)]

In [2.4], the BIH reported on three seven-parameter coordinate transformations between the NSWC 9Z-2 Coordinate System and the three VLBI Coordinate Systems (NGS, NASA/GSFC, and Jet Propulsion Laboratory) associated with the three VLBI data sets used in the analysis. The BIH analysis indicated [2.4; Table 4, Page D-82] that the NSWC 9Z-2 zero meridian should be rotated westward through longitude angles of 0.8079, 0.8126, or 0.8243 arc second, respectively, to achieve agreement with the VLBI zero meridians associated with the NGS, NASA/GSFC, and Jet Propulsion Laboratory (JPL) VLBI data sets.

The preceding NGS and BIH determinations of the relationship between the NSWC 9Z-2 and VLBI reference longitudes contribute to the definition of the longitude angle between the NSWC 9Z-2 zero meridian and the BIH-defined Zero Meridian if the relationship(s) between the VLBI zero meridian(s) and the BIH-defined Zero Meridian is known. The BIH, in [2.5; Table 3, Page B-6], indicates that the BIH-defined Zero Meridian and the reference longitudes for the NGS and JPL VLBI Coordinate Systems are in close agreement; the two VLBI Coordinate Systems requiring westward longitude rotations of only 0.0057 and 0.0078 arc second, respectively, to achieve coincidence with the BIH-defined Zero Meridian. Further, in [2.5; Table 3, Page B-6], the BIH determined that a westward longitude rotation of 0.8137 arc second was needed to bring the NSWC 9Z-2 zero meridian into agreement with the BIH-defined Zero Meridian. Also, in [2.5, Page A-1], the BIH defined the Conventional Terrestrial System (CTS), referring to it as the BIH Terrestrial System (BTS).

Based on the preceding, a westward rotation of 0.814 arc second was adopted by NGS and DMA for bringing the NSWC 9Z-2 X-axis into agreement with the BIH-defined Zero Meridian (1984.0). By adopting this value, a rounded version of the BIH-derived NSWC 9Z-2 longitude rotation angle of 0.8137 arc second appearing in [2.5; Table 3, Page B-6], NGS and DMA establish the BIH-defined Zero Meridian (1984.0) as the reference longitude (zero meridian) for both NAD 83 and WGS 84. However, due to the VLBI-related nature of the 0.814 arc second longitude rotation value, the final word on its validity awaits an accurate relating of the optical and radio (VLBI) reference frames. The relationship between the optical and VLBI reference longitudes is estimated to be known to only ±0.2 arc second [2.8], and it may be difficult to reduce this uncertainty unless an alternative to current astrometric limitations is found. Initial efforts to accurately relate the optical and VLBI reference longitudes [2.9] - [2.12] need to be continued toward a successful conclusion. It is important for astronomic longitudes and

WGS 84 geodetic longitudes to be measured from the same accurately defined reference (zero) meridian.

For completeness, it is noted that there is, practically speaking, no difference between the NSWC 9Z-2 Coordinate System and either the VLBI or SLR Coordinate Systems with respect to rotations about axes (X, Y) in the plane of the earth's equator [2.3] [2.5; Table 3, Page B-6] [2.6; Table 3, Page B-5]. More importantly, this statement is also true for the NSWC 9Z-2 Coordinate System with respect to the BIH-defined Conventional Terrestrial System [2.5; Table 3, Page B-6] [2.6; Table 3, Page B-5]. Therefore, the NSWC 9Z-2 Coordinate System was not rotated about its X or Y axis as part of the WGS 84 development process.

In summary, the best estimate of the difference ($\Delta\lambda$) between the NSWC 9Z-2 Coordinate System reference longitude and the BIH-defined Zero Meridian (1984.0) was taken by NGS and DMA to be:

$$\Delta\lambda = 0.814'' \quad . \tag{2-2}$$

(A one-sigma uncertainty of ±0.2" was assigned to this $\Delta\lambda$ value by DMA.) To implement this longitude modification, the value $\Delta\lambda = 0.814''$ is added to east geodetic longitudes (i.e., longitudes measured eastward from 0° to 360°) that have been derived in the NSWC 9Z-2 Coordinate System.

#### 2.2.2.4 Scale Modification

The scale of the NSWC 9Z-2 Coordinate System is based on the earth's gravitational constant (GM) and the speed of light (c). By virtue of Kepler's Third Law for elliptical motion [2.13], system scale is influenced by the GM value used in the orbit computations [2.14] [2.15]. The velocity of light affects system scale through its use in the conversion of Doppler data to range difference and/or range rate data. These two parameters, GM and c, plus any other scale-related unknown or unmodeled system bias (e.g., displacement of satellite antenna from satellite center of mass), establish the scale of the NSWC 9Z-2 Coordinate System.

A change in GM, for example, produces a change in the geodetic heights of Doppler stations. The validity of such changes to Doppler-derived geodetic heights may be evaluated on specific baselines by comparing Doppler-derived chord distances with VLBI-derived chord distances, SLR-derived chord distances, and chord distances determined from terrestrial survey data [2.3] [2.4].

When the NWL 9D Coordinate System was developed, the value used for the earth's gravitational constant was GM = 398601 x $10^9$ $m^3$ $s^{-2}$, the displacement of the satellite antenna from the center of mass of the satellite was neglected, and the velocity of light used in the computations was c = 299792.5 x $10^3$ m $s^{-1}$. The effect on Doppler station coordinates of using this GM value (rather than GM = 398600.5 x $10^9$ $m^3$ $s^{-2}$), and ignoring the satellite antenna-to-satellite center of mass displacement, was to produce geodetic heights that were 1.7 and 0.7 meters too high, respectively [2.14]. These two results, when combined, indicate that the Doppler-derived (NSWC 9Z-2) geodetic heights of tracking stations should be changed by -2.4 meters. This is equivalent to a change in the NSWC 9Z-2 Coordinate System scale of approximately -0.37 x $10^{-6}$.

The most thorough and extensive analysis of the NSWC 9Z-2 Coordinate System scale is that conducted by NGS as part of their NAD 83 development effort. The NGS analysis [2.3] indicated that the Doppler Coordinate System scale was too large, with respect to the VLBI Coordinate System scale, by scale differences ($\Delta S$) ranging from $\Delta S$ = -0.37 x $10^{-6}$ to $\Delta S$ = -0.69 x $10^{-6}$. A more judicious selection of the data used in the analysis reduced the range of the scale differences, producing values ranging from $\Delta S$ = -0.53 x $10^{-6}$ to $\Delta S$ = -0.69 x $10^{-6}$. The scale difference, $\Delta S$ = -0.6 x $10^{-6}$, was adopted by DMA and NGS as the scale modification to apply to the NSWC 9Z-2 Coordinate System in the development of WGS 84 and NAD 83, respectively.

It is instructive to also review results from some additional investigations of the NSWC 9Z-2 Coordinate System scale. In [2.16], the difference in scale between the NSWC 9Z-2 and AGD 84 Coordinate Systems of $\Delta S = -0.70 \times 10^{-6}$ is worth noting since a related statement affirms that there is no apparent evidence of a scale difference between the terrestrial (AGD 84) and VLBI data. This value agrees quite well with the NGS-derived value of $\Delta S = -0.69 \times 10^{-6}$ discussed above. In [2.4], the BIH reported scale differences between the NSWC 9Z-2 Coordinate System and VLBI Coordinate Systems (NGS; NASA/GSFC) of $\Delta S = -0.57 \times 10^{-6}$ and $\Delta S = -0.49 \times 10^{-6}$, respectively. In a following publication [2.5; Table 3, Page B-6], the BIH identified the scale difference between the NSWC 9Z-2 Coordinate System and the BIH-defined CTS, or BTS, as $\Delta S = -0.604 \times 10^{-6}$. This is significant since a WGS 84 development objective was to have (achieve) agreement between the WGS 84 Coordinate System and the BIH-defined CTS, and this finding supports numerically the DMA and NGS decision to modify the NSWC 9Z-2 Coordinate System in scale by $\Delta S = -0.60 \times 10^{-6}$ (when developing WGS 84 and NAD 83). This feeling of satisfaction is enhanced by noting in the same publication [2.5; Table 3, Page B-6] the good agreement between the BTS and VLBI Coordinate System scales. The BTS scale is larger than the VLBI (NGS) Coordinate System scale by only $\Delta S = 0.044 \times 10^{-6}$, smaller than the VLBI (JPL) Coordinate System scale by only $\Delta S = -0.028 \times 10^{-6}$, and larger than the scale of another VLBI (JPL) Coordinate System by only $\Delta S = 0.048 \times 10^{-6}$. For convenience, candidate scale corrections from the above discussion are listed in Table 2.2.

In summary, the best estimate of the NSWC 9Z-2 Coordinate System scale correction was taken by DMA and NGS to be:

$$\Delta S = -0.6 \times 10^{-6} \quad . \qquad (2\text{-}3)$$

(A one-sigma uncertainty of $\pm 0.1 \times 10^{-6}$ was assigned to this $\Delta S$-value by DMA.) To implement this scale modification <u>at Doppler stations</u> in the NSWC 9Z-2 reference frame, a correction of -3.8 meters is applied to either the geodetic heights (H) or geocentric radii (r). However, some words of caution are required. Although the scale correction $\Delta S = -0.6 \times 10^{-6}$ may be applied on or near the earth's surface and at satellite altitude, the coordinate correction $\delta H \simeq \Delta r = -3.8$ meters is applicable only on or near the earth's surface. This occurs because the value was computed by multiplying the semimajor axis of the WGS 66 Ellipsoid associated with the NSWC 9Z-2 Coordinate System by the scale correction, that is:

$$\begin{aligned} \delta H \simeq \Delta r &= (a) \times \Delta S \\ &= (6378145 \text{ m}) \times (-0.6 \times 10^{-6}) \\ \delta H \simeq \Delta r &= -3.8 \text{ m} \quad . \end{aligned} \tag{2-4}$$

Due to the smallness of the scale change ($\Delta S$), the calculation and treatment of the scale correction in this manner is adequate for earth and near earth applications.

2.2.2.5 Summary/Comments

From Sections 2.2.2.1 - 2.2.2.4, it was concluded that the NSWC 9Z-2 Coordinate System could be improved by:

- Lowering (moving south) the NSWC 9Z-2 origin by 4.5 meters. This is accomplished by adding 4.5 meters to NSWC 9Z-2 Z-coordinates.

- Rotating the NSWC 9Z-2 Reference Meridian (Zero Meridian = X-axis) westward by 0.814 arc second. This is accomplished by adding 0.814 arc second to NSWC 9Z-2 east geodetic longitudes.

- Changing the NSWC 9Z-2 scale by $-0.6 \times 10^{-6}$. This is accomplished by correcting either the geocentric radius or the geodetic height of

Doppler stations in the NSWC 9Z-2 reference frame by -3.8 meters.

The NSWC 9Z-2 Coordinate System, modified in the above manner, becomes (forms) the WGS 84 Coordinate System. These modifications, illustrated individually in Figures 2.1, 2.2, and 2.3, are also summarized in Table 2.3. Insertion of the $\Delta Z$, $\Delta\lambda$, and $\Delta r$ values in the Abridged Molodensky Datum Transformation Formulas, after slightly modifying the formulas and setting $\Delta X = \Delta Y = 0$, provided the $\Delta\phi$, $\Delta\lambda$, $\Delta H$ Formulas (Table 2.4) that produced the Doppler station WGS 84 coordinates used in Chapter 7 to develop Local Geodetic System-to-WGS 84 Datum Transformations.

Further explanation of the scale-related $\Delta r$ value is needed before proceeding. In Figure 2.3, $P_1$ and $P_2$ represent points on the earth's topographic surface whose <u>true</u> positions with respect to each other and the earth's center of mass are invariant with respect to both the NSWC 9Z-2 and WGS 84 Coordinate Systems. However, modification of the Doppler system scale leads in the WGS 84 Coordinate System to an improved mathematical approximation of the <u>true</u> arc or chord distance between the two points and the <u>true</u> length of each point's geocentric radius (r). As seen from Figure 2.3, a decrease $\Delta s$ in the distance $s_{NSWC\ 9Z-2}$ (or decrease $\Delta r$ in the geocentric radii) due to the change in the Doppler system scale leads to the shorter distance $s_{WGS\ 84}$ (or shorter geocentric radii). From this depiction of mathematical compression (the inward movement of Points $P_1$ and $P_2$), it is intuitively apparent that a positive NSWC 9Z-2 geodetic height (H) for Point $P_1$ or $P_2$ would also be decreased by an amount $\delta H \simeq \Delta r$, if the difference between taking H along the geocentric radius instead of the geodetic normal is ignored. Analogously, a negative NSWC 9Z-2 geodetic height for Point $P_1$ or $P_2$ would be increased by an amount $\delta H \simeq \Delta r$. This explanation and Figure 2.3 attempt to describe geometrically how the change in the scale of the NSWC 9Z-2 Coordinate System:

- Has affected the geodetic heights of Doppler stations.

- May be applied to the geodetic heights of Doppler stations as a correction $\delta H \simeq \Delta r$. (See the Equation for $\Delta H$, Table 2.4.)

## 2.3 The WGS 84 Coordinate System

The purpose of much of the preceding has been to establish that the WGS 84 Coordinate System is a Conventional Terrestrial System realized by modifying the NSWC 9Z-2 Coordinate System in origin and scale, and rotating it to bring its reference meridian into coincidence with the BIH-defined Zero Meridian. Thus, as expressed earlier in Section 2.2.1, the origin of the WGS 84 Coordinate System is the center of mass of the earth; the WGS 84 Z-axis is parallel to the direction of the CTP for polar motion, as defined by the BIH on the basis of the coordinates adopted for the BIH stations; the X-axis is the intersection of the WGS 84 reference meridian plane and the plane of the CTP's equator, the reference meridian being parallel to the Zero Meridian defined by the BIH on the basis of the coordinates adopted for the BIH stations; and, the Y-axis, measured in the plane of the above equator, 90° east of the X-axis, completes a right-handed, earth-fixed, orthogonal coordinate system.

The WGS 84 Coordinate System origin and axes also serve as the geometric center and the X, Y, and Z axes of the WGS 84 Ellipsoid. (Thus, the WGS 84 Coordinate System Z-axis is the rotational axis of the WGS 84 Ellipsoid.)

The Doppler satellite coordinate system rather than that of some other space technique (satellite laser ranging, lunar laser ranging, VLBI) was used in developing the WGS 84 Coordinate System since:

- One of the principal purposes of a WGS is to provide the means whereby local geodetic systems can be referenced to a single geocentric system. To accomplish this, both local geodetic system and WGS coordinates are required at one or more sites within the local datum area so that local geodetic system-to-WGS datum shifts can be formed.

- Geodetic positioning of sites of interest was accomplished via satellite point positioning within the Doppler Coordinate System using ephemerides for Navy Navigation Satellite System (NNSS) Satellites and Doppler data acquired at the station (being geocentrically positioned).

- No other space technique can even remotely provide at this time the number and distribution of stations needed to appropriately reference local geodetic systems to a WGS. (A total of 1591 Doppler stations satisfied the editing criteria for use in developing Local Goedetic System-to-WGS 84 Datum Shifts.)

Therefore, the approach selected as most logical for developing the WGS 84 Coordinate System capitalized on the best qualities of the various space techniques, i.e.:

- Proceeded from the NSWC 9Z-2 Coordinate System with its extensive geographic coverage of Doppler stations.

- Utilized the superior ability of the non-Doppler space techniques for providing coordinate system origin, scale, and orientation to obtain data for modifying, improving, and replacing the NSWC 9Z-2 Coordinate System.

As stated previously, the WGS 84 Coordinate System (reference frame) is the frame of a standard earth rotating at a constant rate around an average astronomic pole (the CTP) fixed in time. Therefore, since the universe is in motion, the earth is non-standard, and events occur in an instantaneous world, the WGS 84 Coordinate System (CTS) must be related mathematically to an Instantaneous Terrestrial System (ITS) and an inertial reference frame.

## 2.4 Conventional Inertial System

The concept of a celestial sphere and the definition of some basic planes are necessary for establishing a reference system for earth-centered inertial (ECI) coordinates. The celestial sphere is an imaginary sphere of infinite radius whose center coincides with the center of mass of the earth. The celestial poles and celestial equator are, respectively, projections of the earth's north and south astronomic poles and astronomic equator onto the celestial sphere. The vernal and autumnal equinoxes are the points where the celestial equator intersects the ecliptic (Figure 2.4).

The ecliptic is the plane of the earth's orbit around the sun, or the apparent path of the sun, projected on the celestial sphere. The angle between the celestial equator and the ecliptic, known as the obliquity of the ecliptic, is approximately 23.5 degrees (Figure 2.4).

Since an ideal inertial system experiences neither translation nor rotation, such an ideal system is difficult to realize. The earth is in motion around the sun. Therefore, the origin of the inertial system, the earth's center of mass, experiences translational motion. In addition, the stars are not fixed in space but are also in motion. To establish the stellar-based inertial system to be discussed here, the translational motion of the origin is disregarded and the inertial system is taken to have no rotation with respect to the mean positions of the stars. Such an inertial system, hereafter referred to as a Conventional Inertial System (CIS), is needed to serve as a reference for the earth-fixed ITS and CTS which move with and rotate with the earth.

The FK5 Star Catalog, although not yet available for field use, is available in concept for establishing a CIS. The coordinates (right ascensions, declinations) of the stars in the FK5 Star Catalog define the equator and equinox, and thus the frame, of the FK5 CIS [2.17]. The epoch of the FK5 CIS is the Julian Epoch J2000.0, adopted by the International Astronomical Union (IAU) in 1976 [2.18]. It is this definition of a CIS, expressed pictorially in Figure 2.5, that is used with WGS 84.

## 2.5 Instantaneous Terrestrial System

Due to the gravitational attraction of the sun and moon, the earth's equatorial bulge tries to pull the equatorial plane into the ecliptic plane. Due to the earth's rotation, these attractions produce a conical motion of the earth's axis (true pole) about the pole of the ecliptic plane. This motion, treated in two parts, is in a direction opposite to the earth's rotation and has a period of approximately 26,000 years. One part of the motion, the uniform westward movement of the vernal equinox along the ecliptic, is called luni-solar precession. The other part, called astronomic nutation, is periodic and describes the departure of the actual position of the pole of the celestial equator from its mean position as defined by precession only. Also, there are smaller precessional motions of the ecliptic plane caused by planetary gravitational attraction called planetary precession. The combined effect of luni-solar and planetary precession is called general precession. A new value for the speed of general precession was adopted by the IAU in 1976 [2.18] [2.19].

A new theory of astronomic nutation, referred to as the "1980 IAU Theory of Nutation", has also been adopted by the IAU [2.20]. This new astronomic nutation theory developed by Wahr [2.21] [2.22], based on previous work by Kinoshita, et al. [2.23], and Gilbert and Dziewonski [2.24], consists of two 106-term series instead of the 69-term and 40-term series in the old theory due to Woolard [2.25]. These two series by Wahr provide nutation-induced corrections known as nutation in ecliptic longitude ($\Delta\psi$) and nutation in obliquity ($\Delta\varepsilon$).

The IAU also adopted a new definition of universal time [2.26] which was implemented 1 January 1984 along with the improved value for the general precession of the equinox and new theory of astronomic nutation described above.

As mentioned earlier, the true earth at any instant has an instantaneous astronomic pole and an instantaneous astronomic equator. However, the closest realization of the instantaneous or true astronomic pole for the ITS, that can be established in an astronomical measurement sense, is the Celestial Ephemeris Pole (CEP) or average pole over a 24-hour period. The CEP has no diurnal periodic motion with respect to the earth or space [2.27].

The position (location or movement) of the ITS with respect to the CIS is expressed by the improved value for the general precession of the equinox, the new theory of astronomic nutation, and the new definition of universal time mentioned above. That is, the 1976 IAU constant of precession, the 1980 IAU Theory of Nutation, and the IAU new definition of universal time are used in relating the ITS at some specific time (epoch) to the CIS (Epoch J2000.0).

The motion of the ITS Z-Axis (the CEP) with respect to the CTP is known as polar motion. In Figure 2.6, it is shown that the coordinates of the CEP with respect to the CTP are the polar motion parameters ($x_p$, $y_p$) measured along the 0° and 270° astronomic meridians, respectively. The maximum amplitude of polar motion is approximately 0.3 arc second, corresponding to a displacement (from the CTP) of approximately nine meters on the earth's surface. The BIH publishes polar motion parameters for use in the reduction (referencing) of observations to the CTP.

## 2.6 Mathematical Relationship Between the CIS, ITS, and the WGS 84 Coordinate System (≡BIH-Defined CTS)

The mathematical relationship between the Conventional Inertial System, the Instantaneous Terrestrial System, and the WGS 84 Coordinate System (BIH-defined CTS) can be expressed as:

$$\text{CTS} = [A]\,[B]\,[C]\,[D]\,\text{CIS} \tag{2-5}$$

In Equation (2-5), the rotation matrices for polar motion (A), sidereal time (B), astronomic nutation (C), and precession (D) provide the relationship between the CIS, defined by the FK5 System referenced to Epoch J2000.0, and the WGS 84 Coordinate System (≡BIH-defined CTS). Proceeding from right-to-left in Equation (2-5) through matrices D, C, and B establishes the relationship between the CIS and the ITS. Matrix A provides the relationship between the CEP (which approximates the instantaneous pole of the instantaneous earth) and the CTP, or average pole of the standard earth associated with the WGS 84 Coordinate System (≡BIH-defined CTS). Therefore, the application of Matrix A completes the mathematical connection between the WGS 84 Coordinate System and the CIS.

Downward projections of the CIS pole, the CEP, and the CTP (Figure 2.6) provide pictorially the relationship between the three poles of interest. Figure 2.7 indicates the result achieved through the successive application of the D, C, B, and A rotation matrices.

## 2.7 Summary/Comments

This Chapter has a two fold purpose. One has been to discuss the procedure used to modify and improve the NSWC 9Z-2 Coordinate System to simultaneously form the WGS 84 Coordinate System and establish its commonality with the BIH-defined Conventional Terrestrial System 1984.0 [2.28]. Although considerable effort was expended by the DMA WGS 84 Development Committee analyzing the NSWC 9Z-2 Coordinate System, the DMA-developed $\Delta Z$, $\Delta\lambda$, and $\Delta S$ values were not more definitive than those already published. Therefore, the discussion of these quantities was limited to values available in the literature.

The other purpose has been to discuss the mathematical relationship between a Conventional Inertial System, the ever-present Instantaneous Terrestrial System that requires consideration for many applications, and the WGS 84 Coordinate System (or reference frame) to be used as a reference for DMA-developed MC&G products and data sets. Additional details on the relationships between these systems are provided in the Appendix. Although tremendous progress has been made in the last decade in understanding and more precisely defining the Instantaneous Terrestrial System, the CTS, and the CIS, and the mathematical relationships between them [2.29], much work remains to be done. In particular, efforts to develop a precise mathematical connection between stellar (optical) and radio (VLBI) established CISs and maintain the BIH-defined CTS with respect to a designated epoch must continue.

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Table 2.1

Range of Candidate Z-Axis Bias Values
for
NSWC 9Z-2 Coordinate System

| Source/Reference | Z-Axis Bias ($\Delta Z$) (Meters) |
|---|---|
| NGS [2.3] | 3.60 to 4.08 |
| BIH [2.4] | 4.36 to 5.61 |
| BIH [2.5] | 4.90 |
| BIH [2.6] | 4.73 |
| Selected Z-Axis Bias Value ($\Delta Z$) | $\Delta Z$ = 4.5 Meters |
| Estimated Error ($1\sigma$) | $\sigma_{\Delta Z}$ = ±0.5 Meter |

Table 2.2

Range of Candidate Scale Corrections
for
NSWC 9Z-2 Coordinate System

| References | Scale Correction ($\Delta S$) (Range) | Change in Station Radii ($\Delta r$) |
|---|---|---|
| | ppm | Meters |
| [2.3] | -0.37 to -0.69<br>-0.53 to -0.69 | -2.4 to -4.4<br>-3.4 to -4.4 |
| [2.12] | -0.70 | -4.5 |
| [2.4] | -0.49<br>-0.57 | -3.1<br>-3.6 |
| [2.5] | -0.604 | -3.9 |
| Selected Scale Correction ($\Delta S$) | $\Delta S = -0.6 \times 10^{-6}$ | $\Delta r = -3.8$ m |
| Estimated Error ($1\sigma$) | $\sigma_{\Delta S} = \pm 0.1 \times 10^{-6}$ | $\sigma_{\Delta r} = \pm 0.6$ m |

ppm = parts per million

Table 2.3

Quantities Used in Converting
Doppler System Coordinates (NSWC 9Z-2) To WGS 84*

| Quantities | Explanation | |
|---|---|---|
| $\Delta Z = 4.5$ m | Shift in the Origin (Z-Axis Bias) | Equatorial Plane of Doppler Coordinate System is Offset North of Laser Coordinate System Equatorial Plane |
| $\Delta\lambda = 0.814''$ | Rotation in Longitude | Zero Meridian (X-Axis) of the Doppler Coordinate System is East of the BIH-Defined Zero Meridian (WGS 84 X-Axis) |
| $\Delta S = -0.6 \times 10^{-6}$ | Scale Change | Distances Derived in Doppler Coordinate System are Longer than Distances Determined via Very Long Baseline Interferometry |

* Also, see Table 2.4

Table 2.4

Formulas and Parameters
to Transform NSWC 9Z-2 Coordinates*
to WGS 84 Coordinates

| | |
|---|---|
| Formulas | $\Delta\phi'' = (4.5 \cos\phi) / (a \sin 1'') + (\Delta f \sin 2\phi) / (\sin 1'')$<br>$\Delta\lambda'' = 0.814$<br>$\Delta H_m = 4.5 \sin \phi + a \ \Delta f \ \sin^2\phi - \Delta a + \Delta r$ |
| Parameters | $\Delta f = -0.8120450 \times 10^{-7}$<br>$a = 6378145$ m<br>$\Delta a = -8.0$ m<br>$\Delta r = -3.8$ m (See Figure 2.3) |
| Instructions | To Obtain WGS 84 Coordinates, Add the $\Delta\phi$, $\Delta\lambda$, $\Delta H$ Changes Calculated Using NSWC 9Z-2 Coordinates to the NSWC 9Z-2 Coordinates ($\phi$, $\lambda$, and H, Respectively).<br>[Latitude is Positive North and Longitude is Positive East (0° to 360°)] |

* Navy Navigation Satellite Coordinate System

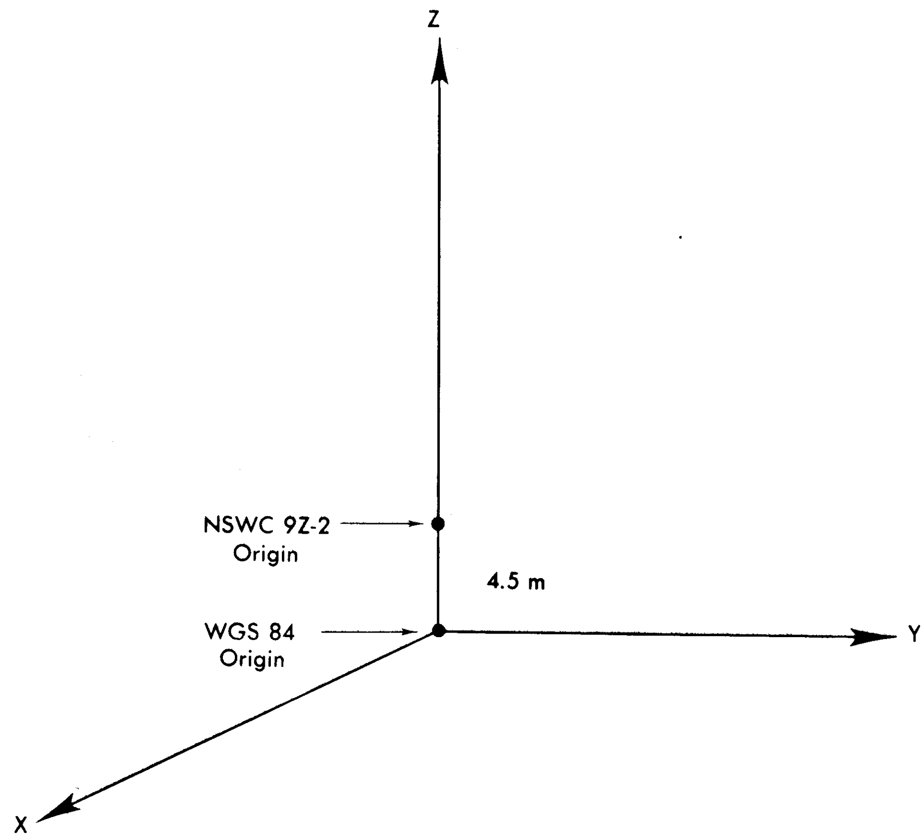



Figure 2.1. Difference Between NSWC 9Z-2 and WGS 84 Reference Frame Origins

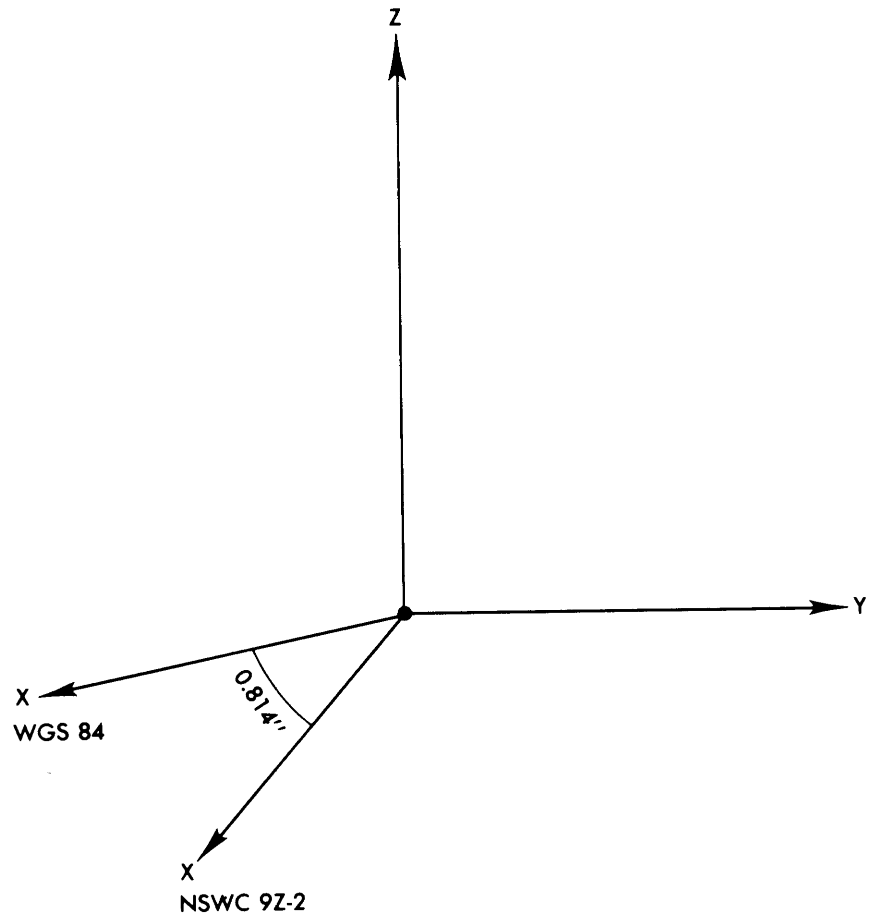
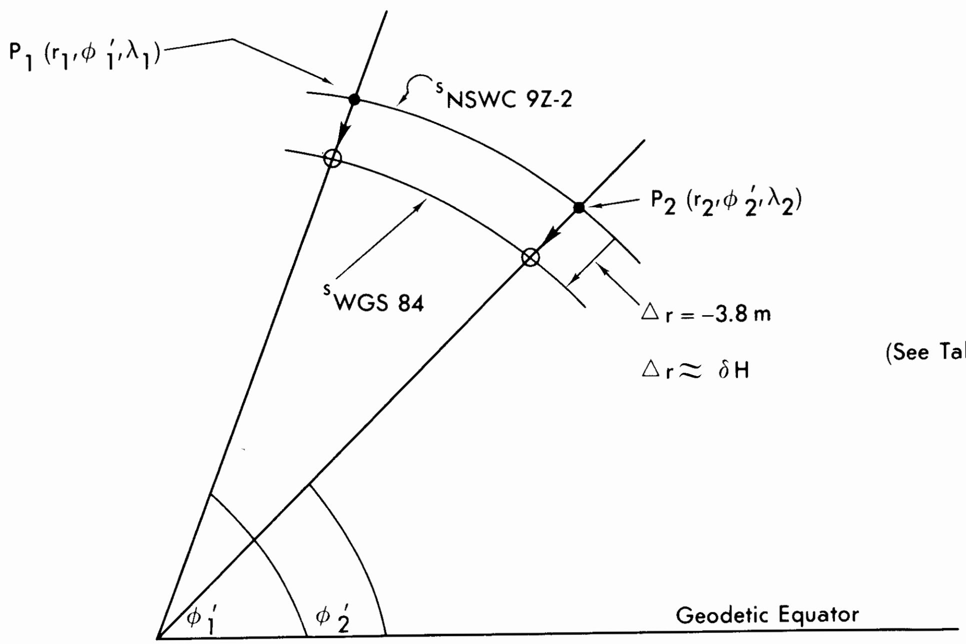



Figure 2.2. Difference Between NSWC 9Z-2 and WGS 84 Longitude References (X-Axes)

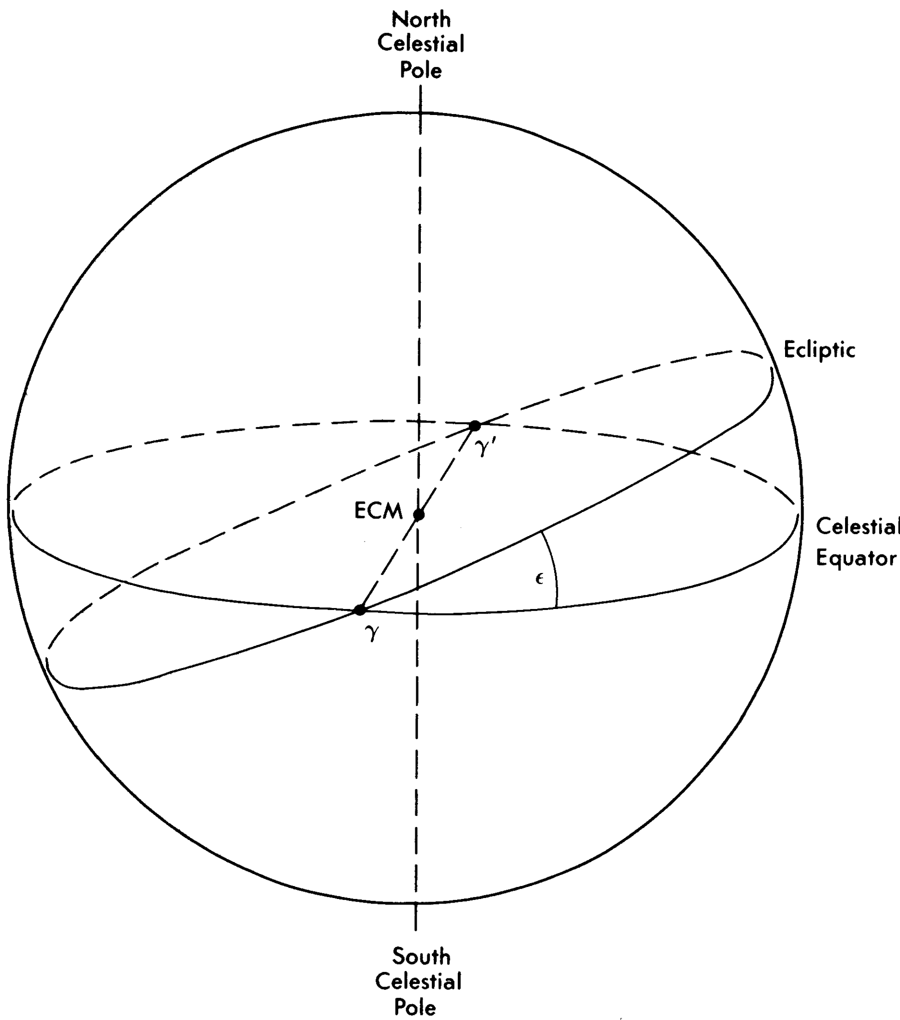
($s_{WGS\ 84} < s_{NSWC\ 9Z\text{-}2}$)

s = distance
r = radius vector
$\phi'$ = geocentric latitude
$\lambda$ = geocentric (geodetic) longitude
H = geodetic height




(See Table 2.4)

Figure 2.3 Differences Between WGS 84 and NSWC 9Z-2 Distances and Geodetic Heights (Effect of Scale Change)




$\gamma$ = Vernal Equinox

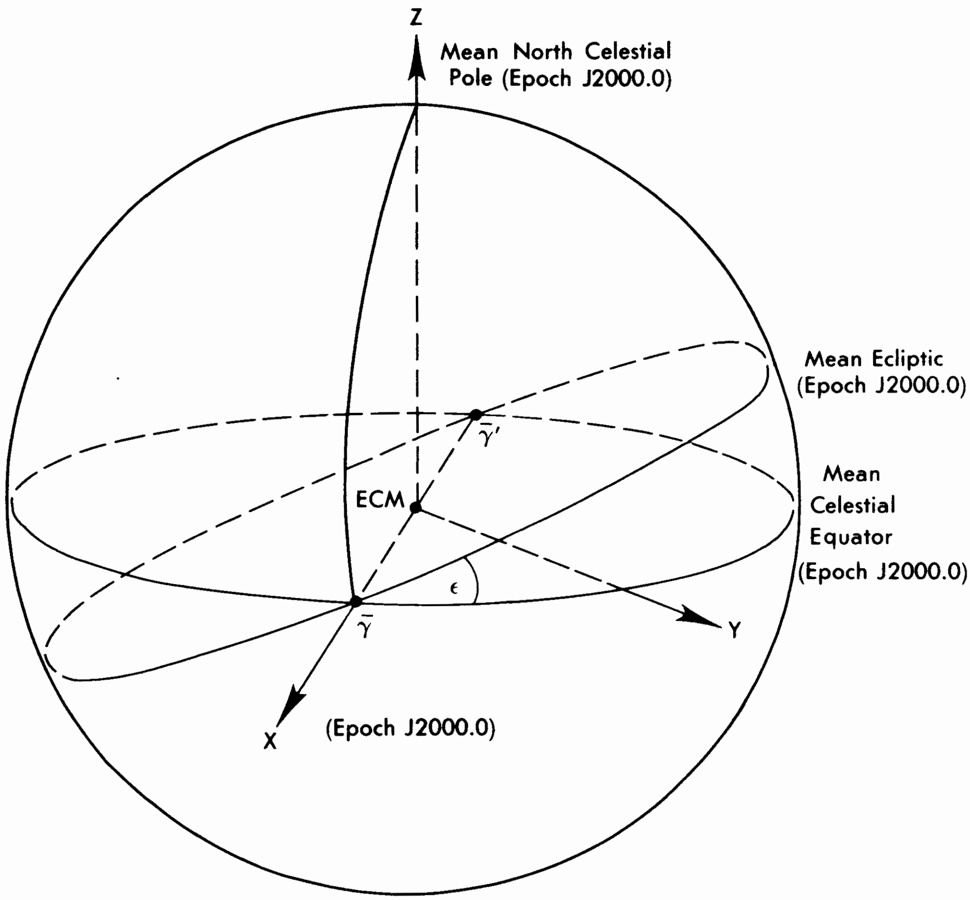
$\gamma'$ = Autumnal Equinox

ECM = Earth's Center of Mass

$\epsilon$ = Obliquity of the Ecliptic

$\epsilon \approx 23°27'$

Figure 2.4. Pictorial Representation of Some Astronomical Terms




$\bar{\gamma}$ = Mean Vernal Equinox (Epoch J2000.0)
$\bar{\gamma}'$ = Mean Autumnal Equinox (Epoch J2000.0)
ECM = Earth's Center of Mass
$\epsilon$ = Obliquity of the Ecliptic
$\epsilon \approxeq 23°27'$

Figure 2.5. Pictorial Definition of the Conventional Inertial System [Stellar (FK5), Epoch J2000.0]

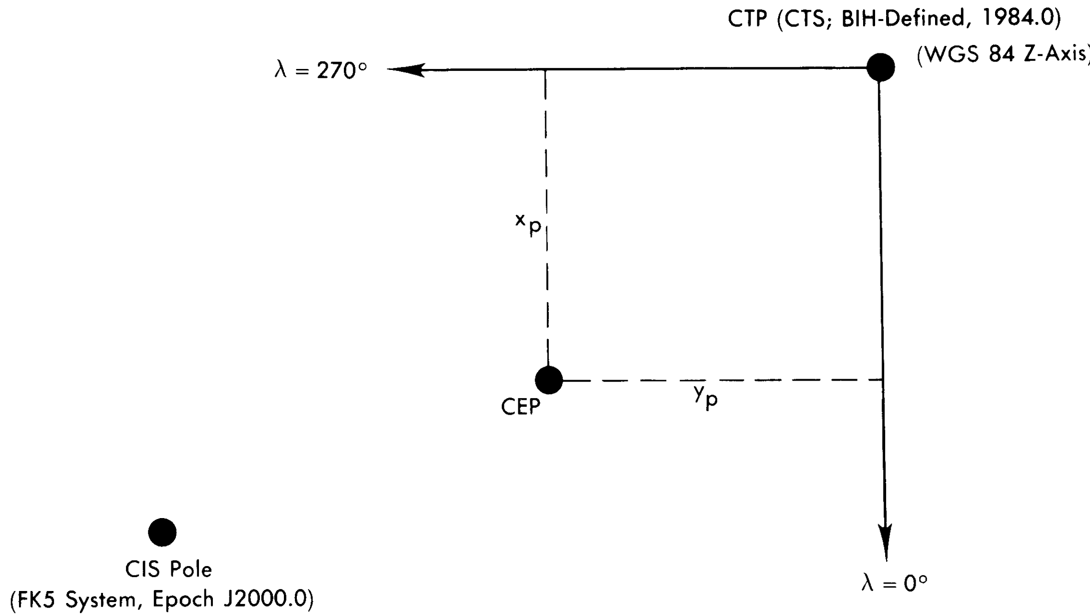
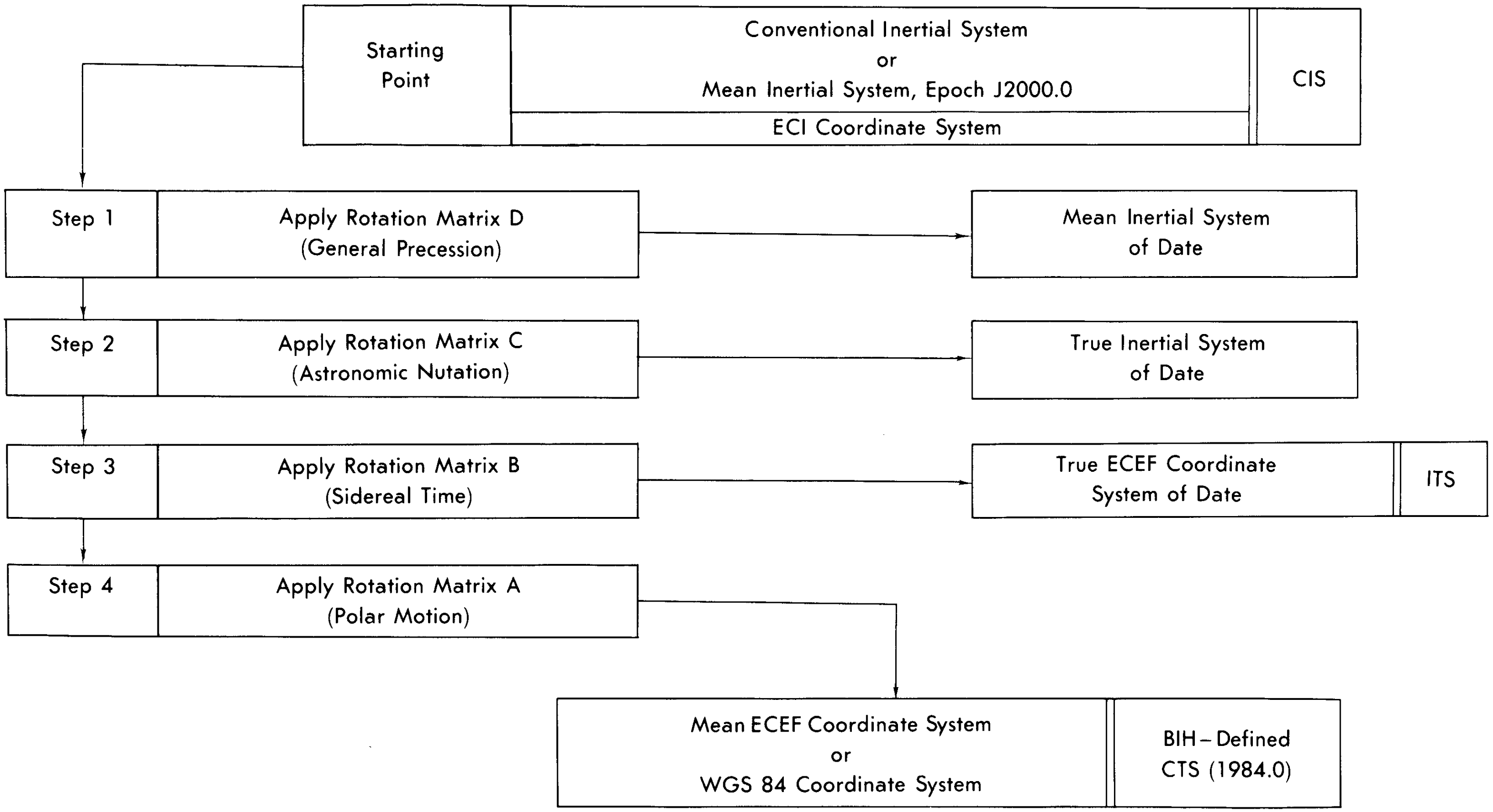



Figure 2.6. Downwardly Projected Conventional Terrestrial Pole (CTP), Celestial Ephemeris Pole (CEP), and Conventional Inertial System (CIS) Pole




[See Appendix.]

Figure 2.7. CIS-to-WGS 84 (CIS-to-CTS) Transformation

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